Math 308R: Bridge to Advanced Mathematics

Homework #11

Due date: Tuesday December 6, 2016, 3:30PM

1. Browse through all of the previous homework sets and the recommended review exercises for the final exam on the course webpage. Find at least 10 problems that are useful for you to review as preparation for the final exam. Make a table listing these problems, as follows:

(a) List the numbers of the 10 (or more) problems in 10 rows.

(b) For each problem in your list, give one or two keywords to indicate what the problem is about.

For example, the beginning of your table could look like this:

Problem	Keywords
CPZ 2.68	Quantifiers, Negation
CPZ 4.79	Contrapositive
HW#6.2	Power set, Induction
...	...

Note: this exercise does not have a right or wrong answer, as long as you find 10 or more problems that you think are useful for you to review.

2.

(a) Find a monic polynomial with integer coefficients of degree 6 which has $\sqrt[3]{\sqrt{6}+2}$ as a solution.

(b) Prove that $\sqrt[3]{\sqrt{6}+2}$ is irrational.

3.

(a) List the four *possible* rational solutions of $3x^3 - 11x^2 - x + 1 = 0$ according to the Rational Root Theorem.

(b) Prove that the equation $3x^3 - 11x^2 - x + 1 = 0$ has exactly one rational solution.

4. Consider the following two subsets of $\mathbb{R}$:
$S := \{3 - \frac{1}{2n} : n \in \mathbb{N}\}$ and $T := \{q^2 : q \in \mathbb{Q} \text{ and } q > 0\}$.

(a) Calculate $\sup S$, $\inf S$, $\sup T$ and $\inf T$.

(b) Which of the numbers calculated in (a) are a maximum or a minimum?

5. Prove or disprove:

(a) For any non-empty bounded subset S of $\mathbb{Q}$, $\sup S$ is rational.

(b) For any non-empty subset S of $\mathbb{Q}$, if S has a maximum, then $\max S$ is rational.