

Math 308R: Bridge to Advanced Mathematics

Homework #5

Due date: Thursday October 13, 2016, 3:30PM

1. Consider the following open statement $P(x)$:

$$\frac{x^2 - 1}{x - 1} = x + 1.$$

- (a) Disprove the statement: $\forall x \in \mathbb{R}, P(x)$.
- (b) Give a domain S such that $\forall x \in S, P(x)$ is true, and prove it.
- 2.

- (a) Prove that $\sqrt{3}$ is an irrational number.
- (b) Disprove the statement: for every natural number n , \sqrt{n} is an irrational number.

3. Consider the following statement, Q :

For every natural number n , if n is odd, then $3 \mid n + 1$ or $n^2 \equiv 1 \pmod{5}$.

- (a) State the negation of the statement Q .
- (b) Disprove the statement Q .
4. Prove by induction that, for every positive integer n , $3 \mid n^3 - n$.
5. A *triomino* is a shape of the following form:



In the lecture, we proved by induction that every board of $2^n \times 2^n$ squares with one corner removed admits a tiling by triominos.

- (a) For every natural number n , let $T(n)$ be the number of triominos used for the tiling of the board of $2^n \times 2^n$ squares with one corner removed.
- Prove that, for every natural number n , $T(n + 1) = 4T(n) + 1$.
- (b) Prove that, for every natural number n , $T(n) = \frac{4^n - 1}{3}$. (Hint: a board of $2^n \times 2^n$ squares with one corner removed has $4^n - 1$ squares.)
6. The twins Aaron and Brad are famous because one of the two only makes true statements, while the other only makes false statements. At a party, you meet one of the twin brothers. He challenges you to find out what his name is. He is in a hurry, so you can only ask him one question.
- (a) What question should you ask? (Hint: you only need to find out his name, you do not need to find out who is the truth-teller and who is the liar.)
- (b) Prove (by cases and/or by contradiction) that your question in (a) works.