Math 308R: Bridge to Advanced Mathematics

Homework #7

Due date: Tuesday October 25, 2016, 3:30PM

1. Find and describe an open conjecture in mathematics. Answer the following questions about the conjecture that you choose:

(a) What is the statement of the conjecture? Describe it in such a way that your fellow students (and your instructor) can understand.

(b) Who formulated the conjecture, and when?

(c) Has a prize been offered for a proof of the conjecture?

- **Cite** any internet and/or library resources that you use.
- **Examples** of conjectures that you could use are: twin prime conjecture, infinitely many perfect numbers conjecture, Erdös conjecture on arithmetic progressions.

2. Prove or disprove:

(a) For any integers a, b, if $3 \mid ab$ then $3 \mid a$ and $3 \mid b$.

(b) For any integers a, b, if $3 \mid ab$ then $3 \mid a$ or $3 \mid b$.

(c) For any integers a, b, if $4 \mid ab$ then $4 \mid a$ or $4 \mid b$.

(d) (Extra Credit) For every prime number p, for any integers a, b, if $p \mid ab$ then $p \mid a$ or $p \mid b$.

3. For each of the following statements, state the negation of the statement, and disprove the statement (i.e., prove the negation):

(a) For any real numbers x, y, $(x + y)^2 = x^2 + y^2$.

(b) For every integer x, there exists an integer y such that $xy \equiv 1 \pmod 4$.

(c) There exist integers p, q such that $\sqrt{2} = \frac{p}{q}$.

(d) There exists a set A such that for every subset $B \subseteq A$, $|A - B| \geq 1$.

4. Let $A = \{1, 2, 3\}$ and let R be the following relation on A: $R \overset{\text{def}}{=} \{(1, 2), (2, 2), (3, 2), (3, 3)\}$.

(a) Draw the graph of the relation R.

(b) Prove or disprove: R is reflexive.

(c) Prove or disprove: R is symmetric.

(d) Prove or disprove: R is transitive.

(e) Draw the graph of the inverse relation R^{-1}.

(f) Using set notation, list the elements of R^{-1}.