Math 308R: Bridge to Advanced Mathematics

Homework #8

Due date: Tuesday November 1, 2016, 3:30PM

1. The relation S on $\mathbb{R} \times \mathbb{R}$ is defined by

$$(x_1, y_1)S(x_2, y_2) \iff x_1 = x_2.$$

(a) Prove that S is an equivalence relation.

(b) Give three elements of the equivalence class $[(1, 5)]$.

(c) Prove that, for every $y \in \mathbb{R}$, $[(1, 5)] = [(1, y)]$.

(d) Draw the subset $[(1, 5)]$ of $\mathbb{R} \times \mathbb{R}$.

(e) Describe the partition of $\mathbb{R} \times \mathbb{R}$ induced by S.

2. The relation R on $\mathbb{Z}$ is defined by

$$R := \{(a, b) \in \mathbb{Z} \times \mathbb{Z} : a^2 \equiv b^2 \pmod{3}\}.$$

(a) Prove that R is an equivalence relation on $\mathbb{Z}$.

(b) List the elements of the equivalence class $[0]$.

(c) Prove that $[1] = [2]$.

(d) Prove that $[1] = \{b \in \mathbb{Z} : b \equiv 1 \pmod{3} \text{ or } b \equiv 2 \pmod{3}\}$.

3. Using that addition and multiplication are well-defined in $\mathbb{Z}_n$, prove the following:

(a) For any integer x, if $x \equiv 1 \pmod{9}$ then $x^{2016} \equiv 1 \pmod{9}$.

(b) For any integer x, if $6 \mid x - 3$, then $6 \mid x^{100} - 3$.

(c) For any integers x, y, if $5 \mid x - 2$ and $5 \mid y - 4$, then $5 \mid xy - 3$.

4. In this problem you are asked to give examples of relations. In each item, **prove** that the example you give satisfies all the requirements.

(a) Give an example of a relation R from $\mathbb{N}$ to $\mathbb{Q}$ such that $\text{dom}(R) = \{1, 2, 7\}$ and $\text{range}(R) = \{\frac{1}{2}, \frac{9}{5}\}$.

(b) Give an example of a relation S on $\mathbb{N}$ which is reflexive and symmetric, but not transitive.

(c) Give an example of an equivalence relation T on the set $\{1, 2, 3, 4, 5, 6\}$ such that each equivalence class contains exactly 2 elements.