Math 308R: Bridge to Advanced Mathematics

Addendum to class of December 1, 2016

Ross Exercise 2.7(a). Prove that $\sqrt{4+2\sqrt{3}}-\sqrt{3}$ is rational.

Solution. We first do a 'secret calculation'. Let $a := \sqrt{4+2\sqrt{3}}-\sqrt{3}$. Then we get that $a+\sqrt{3}=\sqrt{4+2\sqrt{3}}$. Therefore,

$$\begin{aligned} a^2+3+2a\sqrt{3} &= 4+2\sqrt{3} \\ a^2-1+2\sqrt{3}(a-1) &= 0 \\ (a+1)(a-1)+2\sqrt{3}(a-1) &= 0 \\ (a+1+2\sqrt{3})(a-1) &= 0. \end{aligned}$$

Therefore, $a=1$ or $a=-1-2\sqrt{3}$. Since a is not negative, $a=1$.
We claim that

$$\sqrt{4+2\sqrt{3}}-\sqrt{3}=1,$$

which is rational.
Proof. Note that $4+2\sqrt{3}=(1+\sqrt{3})^2$. Therefore, $\sqrt{4+2\sqrt{3}}=1+\sqrt{3}$. Hence,

$$\sqrt{4+2\sqrt{3}}-\sqrt{3}=(1+\sqrt{3})-\sqrt{3}=1.$$