Spaces with and without points

Duality and canonical extensions for stably compact spaces

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Radboud Universiteit Nijmegen

7 October 2011
WONDER PhD Colloquium

Presentation outline

1. Stone duality
   - Boolean spaces
   - Spectral spaces

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1. Stone duality
   - Boolean spaces
   - Spectral spaces
2. Canonical extensions
   - Duality via canonical extensions
3. Stably compact spaces
   - Extending Stone duality
   - Duality via Karoubi envelope
   - Proximity lattices
   - Main result

Zero-dimensional spaces

Example

Cantor space $X = \{0, 1\}^{\mathbb{N}}$

Zero-dimensional spaces

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The sets $K_{(a_1,\ldots,a_n)}$ form a basis of clopens!

Zero-dimensional spaces

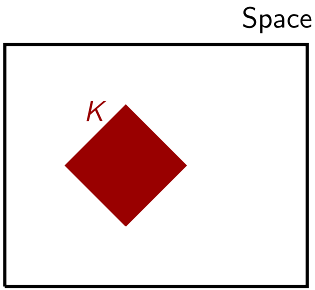
Definition

A space is zero-dimensional if the clopens form a basis for the topology.
A Boolean space is a zero-dimensional compact T_2 space.

Objects

Space

Objects



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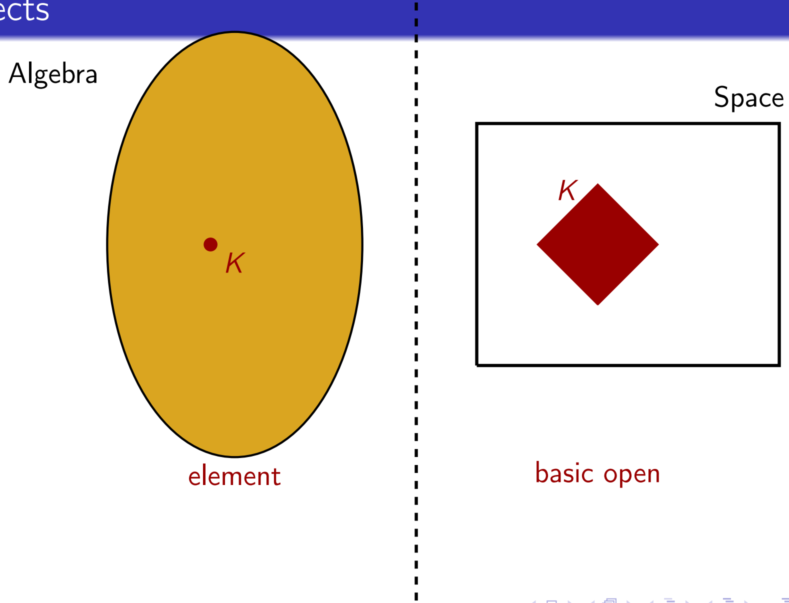

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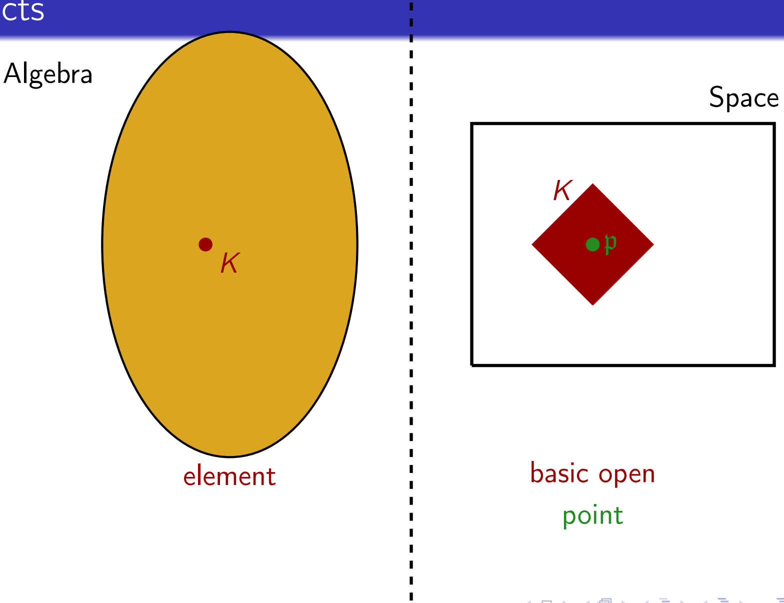

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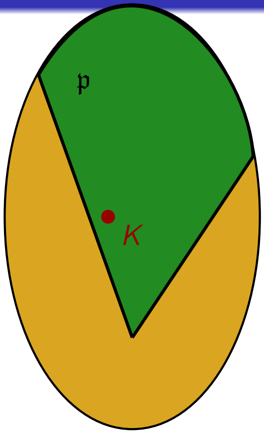

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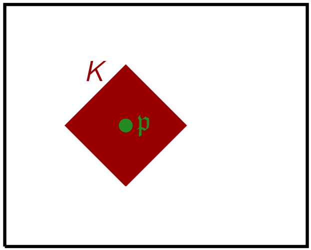

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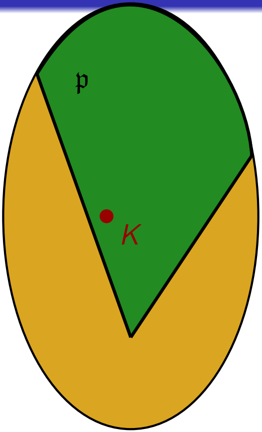

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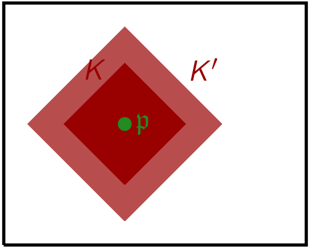

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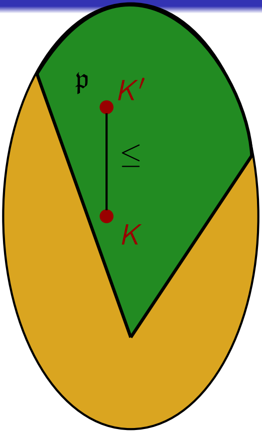

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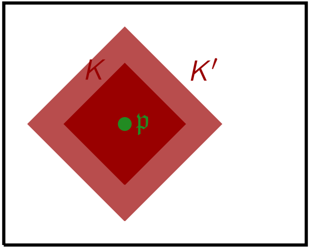

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Morphisms

X

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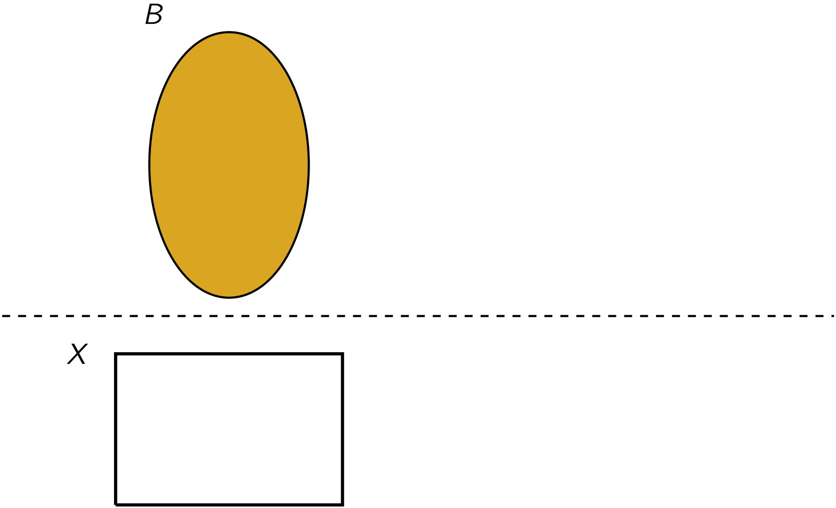

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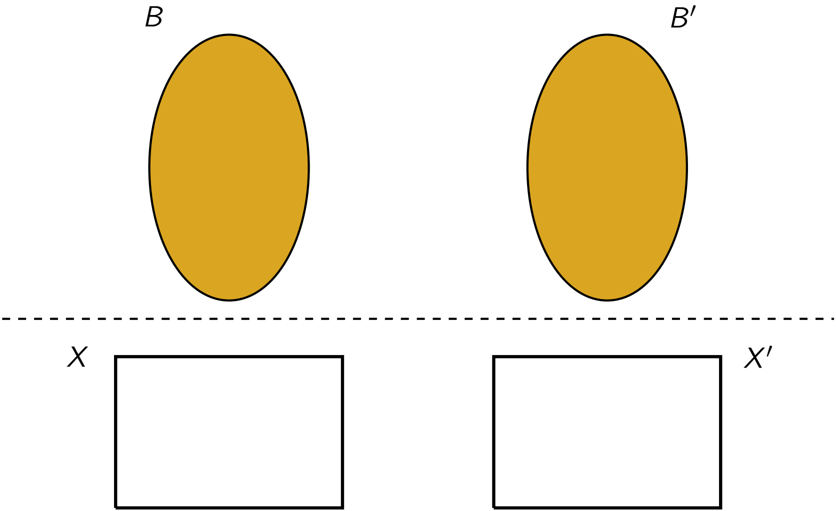

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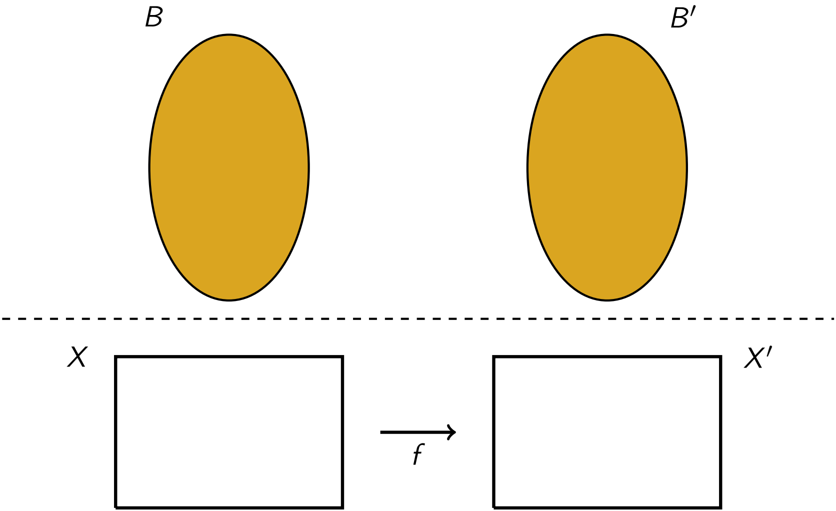

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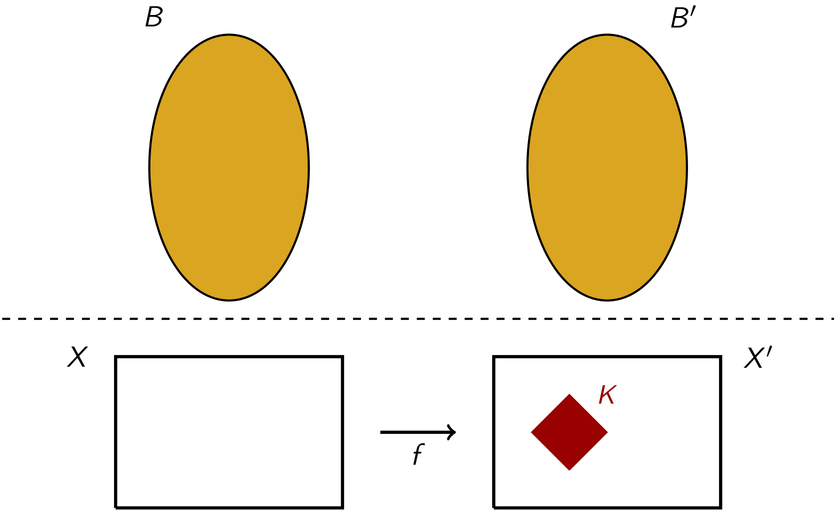

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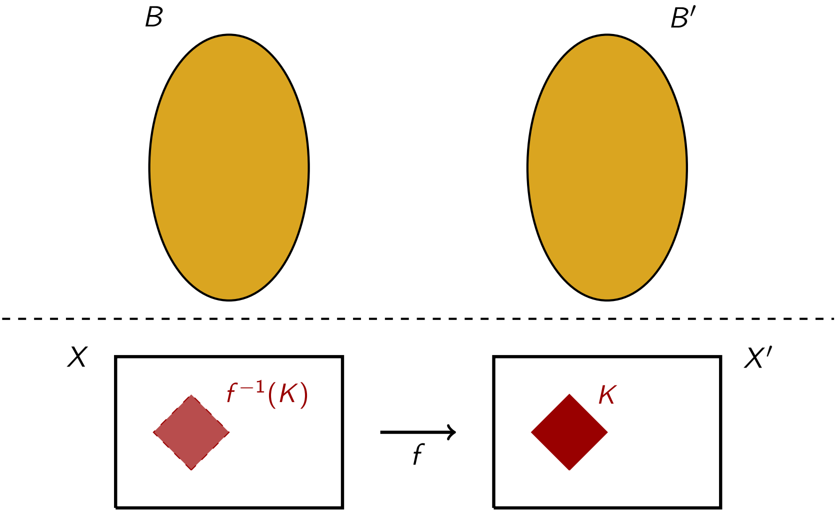

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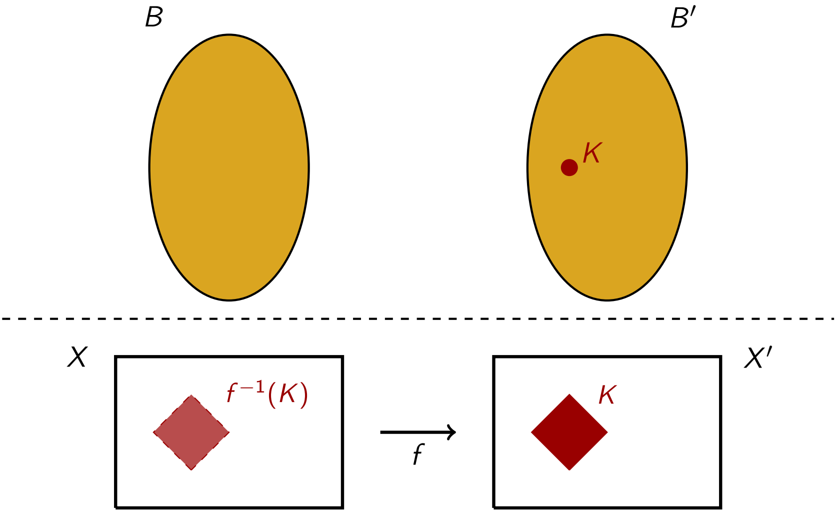

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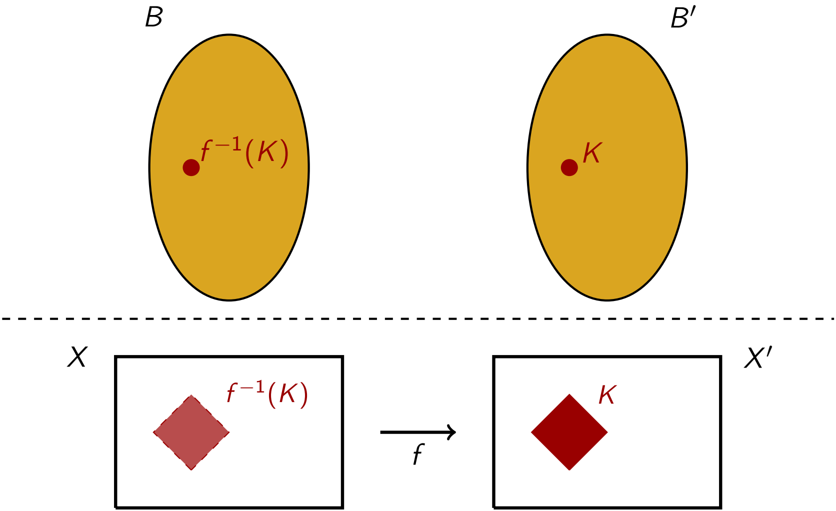

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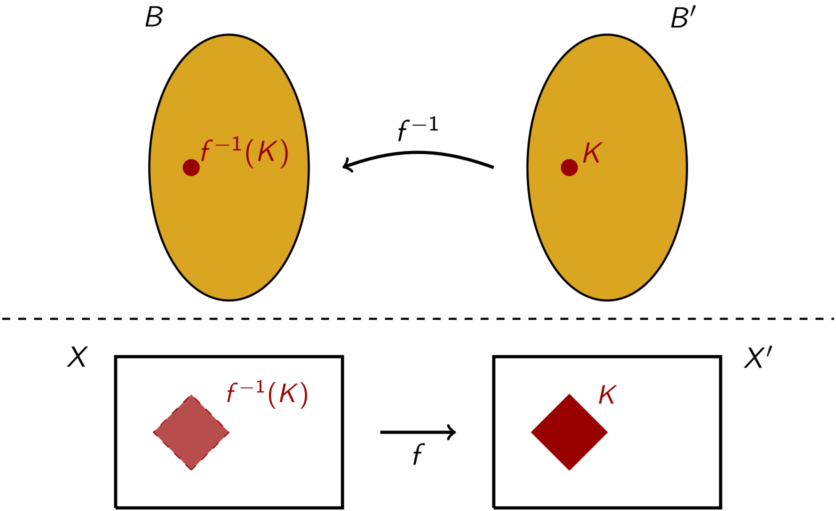

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Theorem (Stone, 1936)

The categories **BoolSp** *and* **BoolAlg** *are dually equivalent.*

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Theorem (Hochster, 1969)

Any spectral space arises as the prime ideal spectrum of some ring.

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Theorem (Stone, 1937)

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- Can be extended to spectral spaces with all continuous functions; dual category of distributive lattices with "j-morphisms" (will return to these later).

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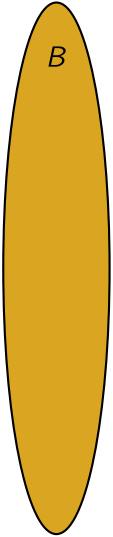
Theorem (Jónsson & Tarski (1951), Gehrke & Jónsson (1994))

The canonical extension of a Boolean algebra (distributive lattice) exists uniquely, and can be viewed as the algebra of subsets (up-sets) of the space X.

Gehrke-Jónsson-Tarski theorem pictorially



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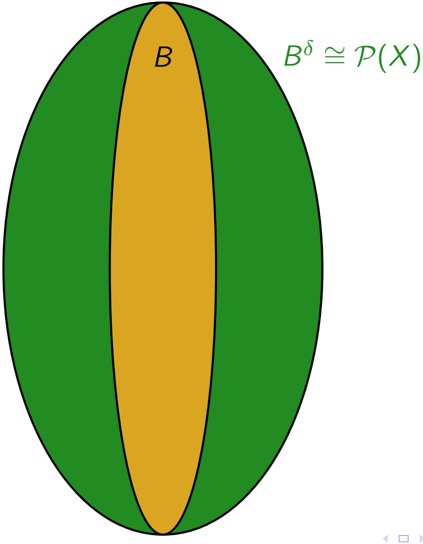

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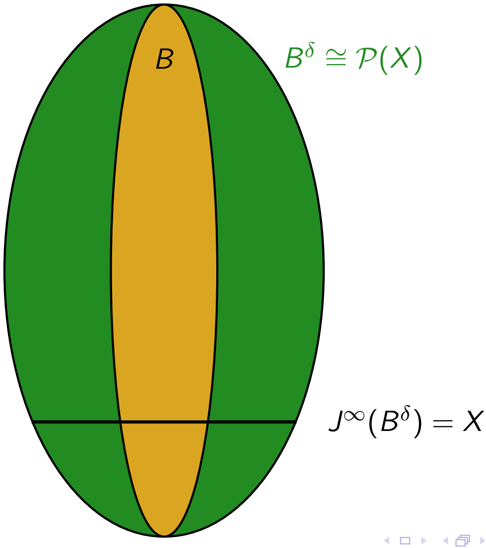

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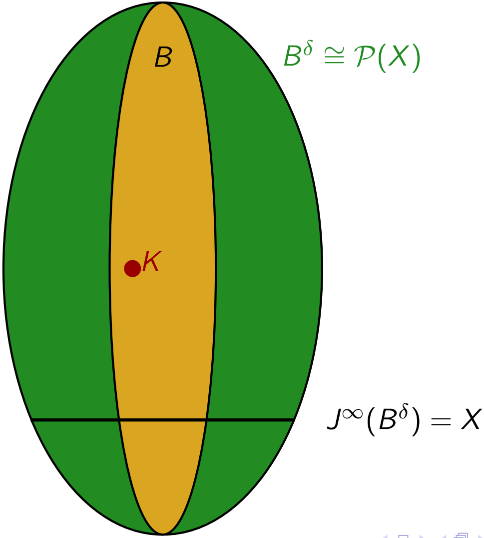

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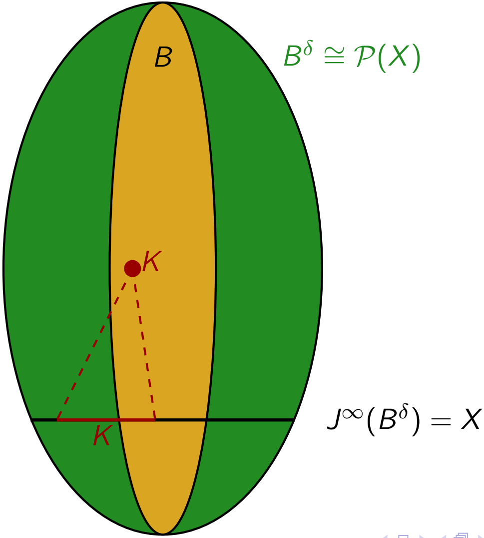

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     - reduces complexity of algebraic representation.

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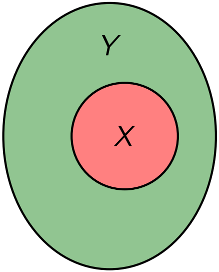
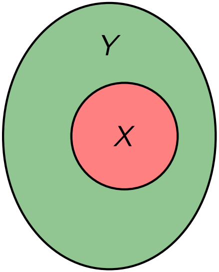
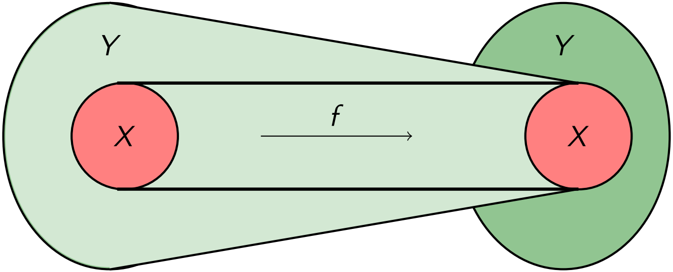




Spectral spaces with retractions

Fact (Johnstone, 1982)

A topological space X is stably compact iff there is a spectral space Y and a continuous retraction $f : Y \to Y$ onto X.



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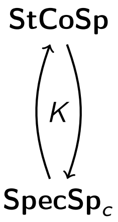
- If $\bigvee A\, H\, b$ then $\exists B \subseteq_\omega H[A]$ such that $b \leq \bigvee B$.

Karoubi envelope lifts spectral duality

- The category **StCoSp** is the Karoubi envelope of $\mathbf{SpecSp}_c$.



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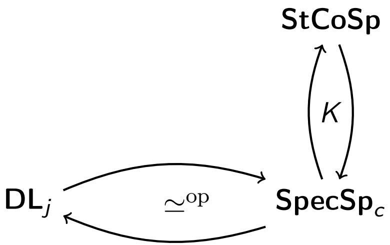

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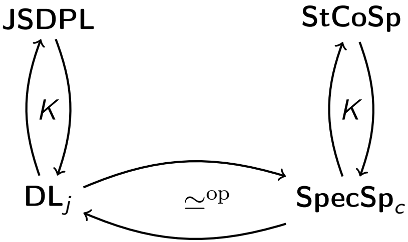
- Karoubi envelope of $\mathbf{DL}_j$: join-strong distributive proximity lattices (Smyth 1992).



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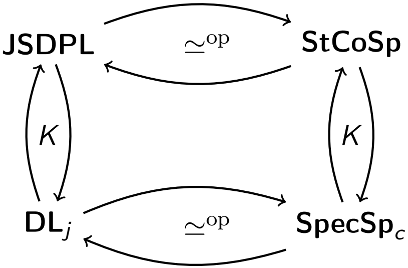

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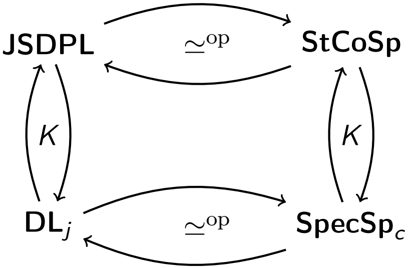

We give a detailed proof of **JSDPL** $\simeq^{\mathrm{op}}$ **StCoSp** using the Karoubi envelope in [vGo2011].

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- $a\,R \bigwedge B$ iff $B \subseteq R[a]$

Canonical extension

Existence and uniqueness

Theorem (SvG, 2011)

Every join-strong proximity lattice (L, R) has a unique π-canonical extension $(L, R)^\pi$. If L is distributive, then $(L, R)^\pi$ is the lattice of saturated sets of the space dual to (L, R).

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- Canonical extensions: method to produce points of Cantor space by algebraic methods.
- Stably compact spaces: retracts of Cantor space dually give retracts of the free algebra, i.e., projective algebras.

Selected References

- [GeJo1994] Mai Gehrke, Bjarni Jónsson, Bounded distributive lattices with operators, Mathematica Japonica.

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Spaces with and without points

Duality and canonical extensions for stably compact spaces

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7 October 2011
WONDER PhD Colloquium